Identifying codes in bipartite graphs of given maximum degree

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joint work with:

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and

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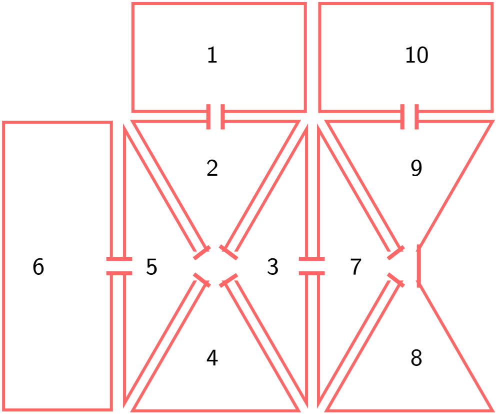
An example: Identifying codes (ID)



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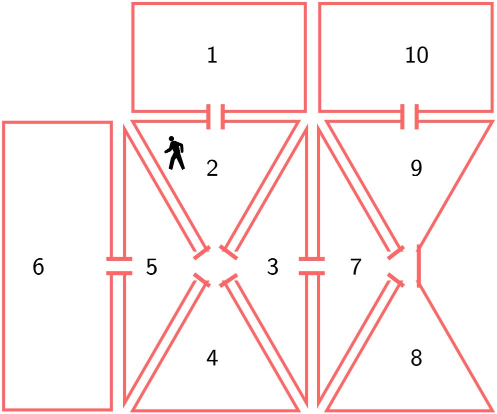

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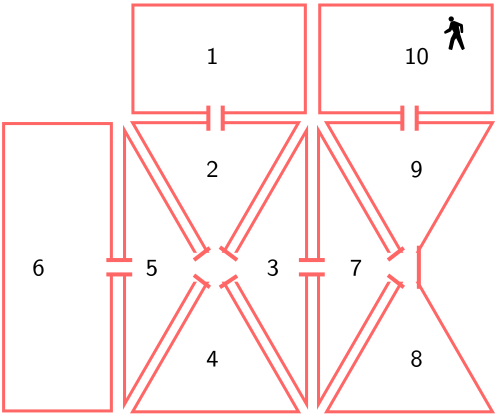

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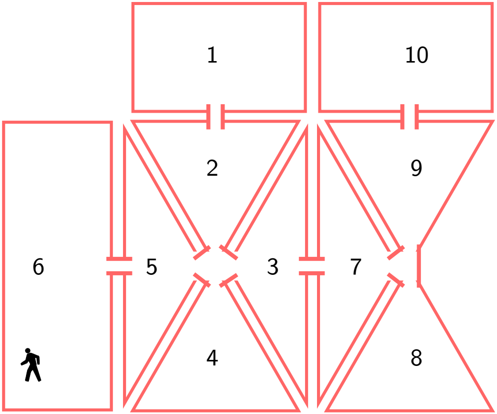

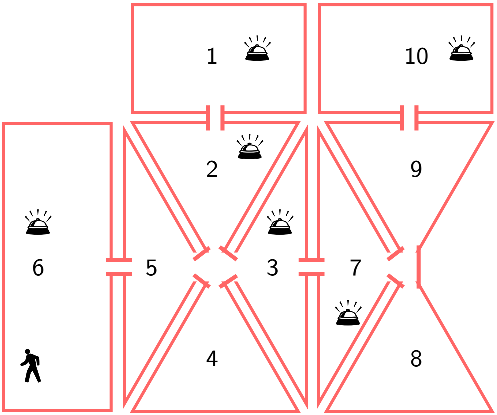
1
10
2
9
6
5
3
7
4
8

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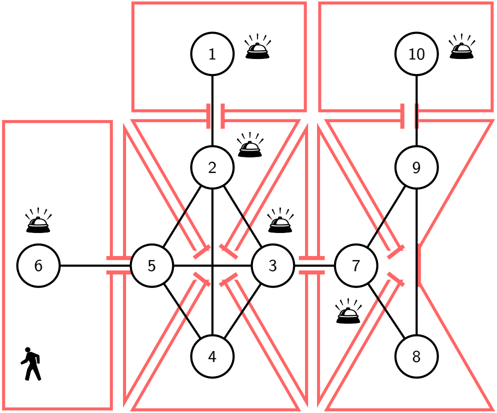

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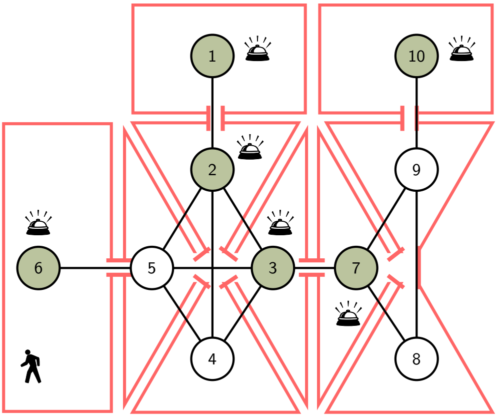

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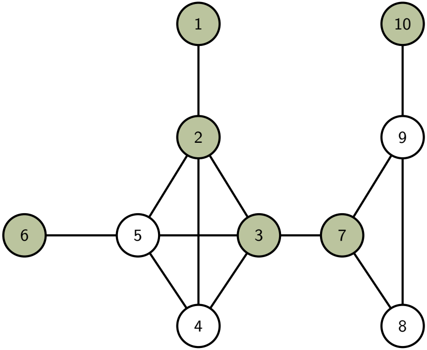

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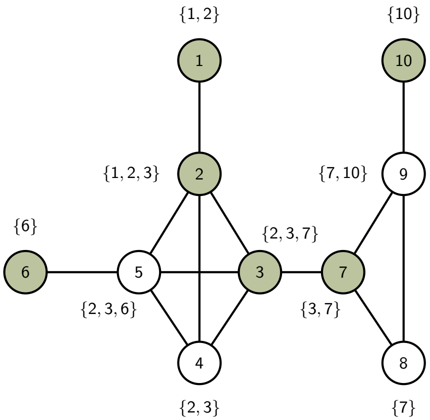

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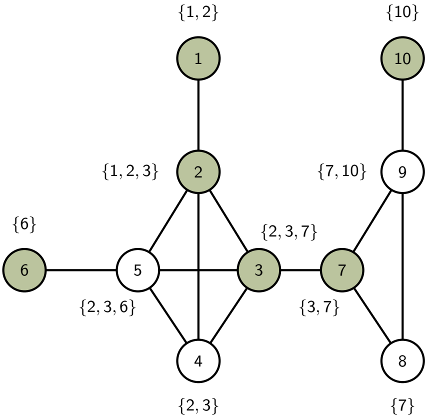

- 1 $\longleftrightarrow$ $\{1, 2\}$
- 2 $\longleftrightarrow$ $\{1, 2, 3\}$
- 3 $\longleftrightarrow$ $\{2, 3, 7\}$
- 4 $\longleftrightarrow$ $\{2, 3\}$
- 5 $\longleftrightarrow$ $\{2, 3, 6\}$
- 6 $\longleftrightarrow$ $\{6\}$
- 7 $\longleftrightarrow$ $\{3, 7\}$
- 8 $\longleftrightarrow$ $\{7\}$
- 9 $\longleftrightarrow$ $\{7, 10\}$
- 10 $\longleftrightarrow$ $\{10\}$

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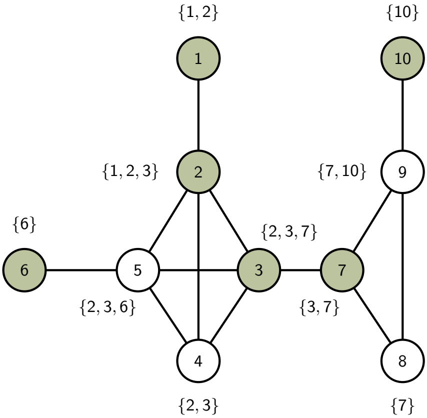

- $1 \longleftrightarrow \{1, 2\} \quad = N[1] \cap C$
- $2 \longleftrightarrow \{1, 2, 3\} = N[2] \cap C$
- $3 \longleftrightarrow \{2, 3, 7\} = N[3] \cap C$
- $4 \longleftrightarrow \{2, 3\} \quad = N[4] \cap C$
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- $9 \longleftrightarrow \{7, 10\} \quad = N[9] \cap C$
- $10 \longleftrightarrow \{10\} \quad = N[10] \cap C$

$C = \{$ "chosen" vertices$\}$

Identifying codes: definition

Let $N[u]$ be the set of vertices v s.t. $d(u, v) \leq 1$

Definition - Identifying code of G (Karpovsky, Chakrabarty, Levitin, 1998)

Subset C of V such that:

- C is a **dominating set** in G: $\forall u \in V$, $N[u] \cap C \neq \emptyset$, and
- C is a **separating code** in G: $\forall u \neq v$ of V, $N[u] \cap C \neq N[v] \cap C$

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Notation - Identifying code number

$\gamma^{\text{ID}}(G)$: minimum cardinality of an identifying code of G

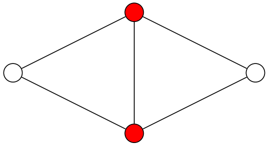
$N[u]$: set of vertices v s.t. $d(u, v) \leq 1$

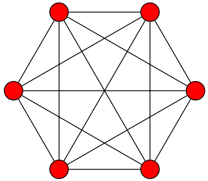
Remark

Not all graphs have an identifying code!

Closed twins = pair u, v such that $N[u] = N[v]$.

A graph is **identifiable** iff it is **closed twin-free** (i.e. it has no closed twins).





Hardness of the decision problem

Definition - ID-CODE (decision problem)

Input: Graph G and an integer $k \leq |V(G)|$.
Question: Is there and ID-code of G of order at most k?

Theorem (Hardness: Charon, Hudry, Lobstein, 2001)

ID-CODE is NP-complete for bipartite graphs.

Theorem (Hardness: Muller, Sereni, 2009)

ID-CODE is NP-complete for planar bipartite unit disk graphs.

Hardness of the decision problem

Theorem (Hardness: Auger, 2009)

ID-Code is NP-complete for planar graphs of arbitrarily large girth.

Theorem (Hardness: Foucaud, 2015)

ID-Code is NP-complete for:

- planar bipartite subcubic graphs.
- Chordal bipartite graphs.

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Theorem (lower bound: Karpovsky, Chakrabarty, Levitin, 1998
upper bound: Bertrand, 2005 / Gravier, Moncel, 2007 / Skaggs, 2007)

Let G be an identifiable graph on n vertices with at least one edge, then

$$\lceil \log_2(n+1) \rceil \leq \gamma^{\text{ID}}(G) \leq n-1$$

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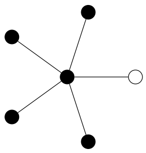

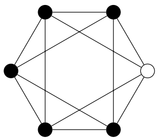

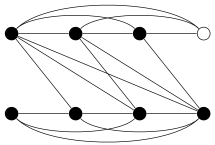

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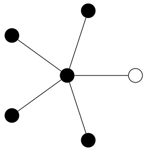

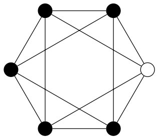

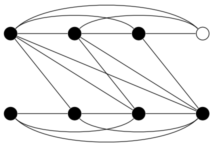

Observation

All these graphs have maximum degree $n-1$ or $n-2$!

A conjecture

Theorem (Karpovsky, Chakrabarty, Levitin, 1998)

Let G be an identifiable graph with maximum degree Δ and n vertices, then

$$\frac{2}{\Delta+2} n \leq \gamma^{\text{ID}}(G)$$

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Conjecture (Foucaud, Klasing, Kosowski, Raspaud, 2009)

Let G be a connected nontrivial identifiable graph on n vertices and of maximum degree Δ. Then:

$$\gamma^{\text{ID}}(G) \leq \frac{\Delta-1}{\Delta}n + c = n - \frac{n}{\Delta} + c \text{ (for some constant } c\text{).}$$

Remark

The conjecture is true for $\Delta = 2$ (with $c = 3/2$)

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Extremal examples

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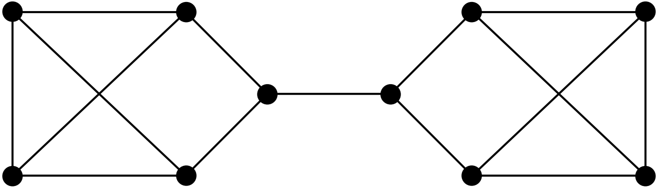

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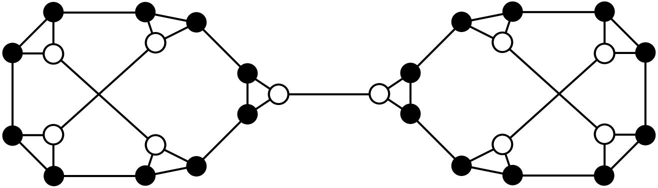

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Also: Sierpiński graphs
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Remark

The bound is tight!

Previous work

Theorem (Foucaud, Perarnau, 2012)

For an identifiable graph G of maximum degree Δ, we have

$$\gamma^{\text{ID}}(G) \leq n - \frac{n}{\Theta(\Delta^3)}; \text{ in the general case.}$$

$$\gamma^{\text{ID}}(G) \leq n - \frac{n}{103\Delta}n; \text{ } G \text{ is regular.}$$

$$\gamma^{\text{ID}}(G) \leq n - \frac{n}{f(k)\Delta}n; \text{ } G \text{ is of clique number at most } k.$$

Previous work

Theorem (Foucaud, Klasing, Kosowski, Raspaud, 2012)

Let G be a connected identifiable triangle-free graph on n vertices with maximum degree $\Delta \geq 3$. Then

$$\gamma^{\text{ID}}(G) \leq n - \frac{n}{\Delta + o(\Delta)};\ \text{in the general case.}$$

$$\gamma^{\text{ID}}(G) \leq n - \frac{n}{\Delta + 9};\ G \text{ is bipartite or planar.}$$

$$\gamma^{\text{ID}}(G) \leq n - \frac{n}{o(\Delta)};\ G \text{ has no open twins.}$$

$$\gamma^{\text{ID}}(G) \leq n - \frac{n}{8} + 1;\ G \text{ has minimum degree 2 and girth at least 5.}$$

Previous work

Theorem (Preceding work: Foucaud, Lehtilä, 2022)

Let G ($\ncong P_4$) be a bipartite graphs without (open) twins on $n \geq 3$ vertices. Then

$$\gamma^{\text{ID}}(G) \leq \frac{2}{3}n \leq \frac{\Delta-1}{\Delta}n \ \text{ for } \Delta \geq 3.$$

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Open question

Does the conjecture hold for trees?

Our work: Bipartite graphs

Theorem (C., Foucaud, Lehtilä, 2023+)

Let G be a connected bipartite graph of order n, of maximum degree $\Delta \geq 3$, with no twins of degree 2 or greater, and not isomorphic to any graph in the collection $\mathcal{F}_\Delta$. Then,we have

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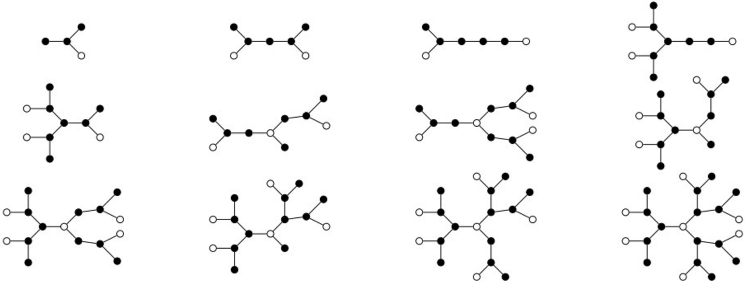
Our class of bipartite graphs includes:

- All trees!
- All bipartite graphs without 4-cycles!

$\mathcal{F}_3 = \mathcal{T}_{\text{trees}}$ and $\mathcal{F}_\Delta = \{K_{1,\Delta}\}$ for $\Delta \geq 4$.



Proposition (C., Foucaud, Lehtilä, 2023+)

For any $G \in \mathcal{F}_\Delta$, we have

$$\gamma^{\text{ID}}(G) \leq \frac{\Delta-1}{\Delta} n + \frac{1}{\Delta}.$$

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Proof sketch

- Induction on the dictionary order on (n, m);
  $n = \#$(vertices of G), $m = \#$(edges of G).

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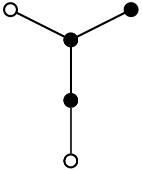

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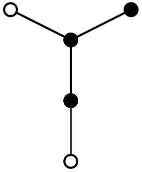
- Induction on the dictionary order on (n, m); $n = \#(\text{vertices of } G)$, $m = \#(\text{edges of } G)$.
- Base case: $(n, m) = (5, 4)$.
- Assume hypothesis on all connected bipartite graphs G' ($\notin \mathcal{F}_\Delta$) with no twins of degree 2 or more and with $(n', m') \geq_d (5, 4)$.



Main theorem: Proof sketch

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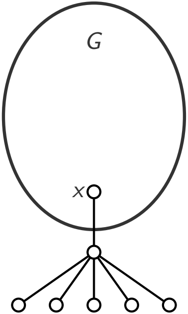
Proof sketch

Lemma (Foucaud, Lehtilä, 2022)

Let $G(\ncong P_4)$ be a connected bipartite graph on $n \geq 4$ vertices, with s support vertices, ℓ leaves and no twins of degree 2 or more. Then, we have

$$\gamma^{ID}(G) \leq n - s \text{ and } \gamma^{ID}(G) \leq \frac{n+\ell}{2}.$$



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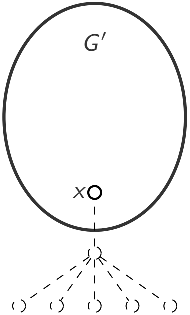

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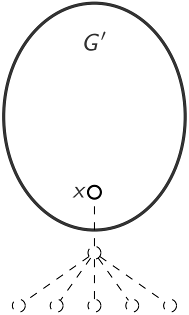

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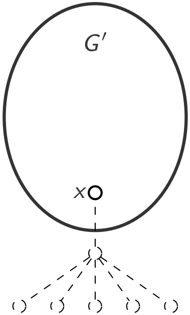

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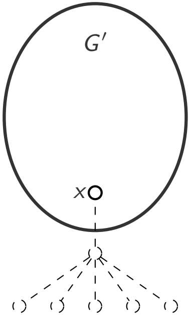

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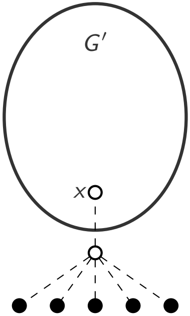

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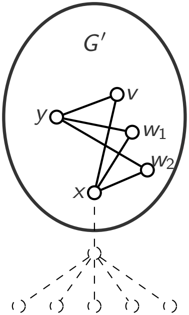

Main theorem: Proof sketch

Theorem (C., Foucaud, Lehtilä, 2023+)

Let G be a connected bipartite graph of order n, of maximum degree $\Delta \geq 3$, with no twins of degree 2 or greater, and not isomorphic to any graph in the collection $\mathcal{F}_\Delta$. Then,we have $\gamma^{\text{ID}}(G) \leq \frac{\Delta-1}{\Delta}n$.

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- Delete the leaves and the support vertex. Call the new graph G'.
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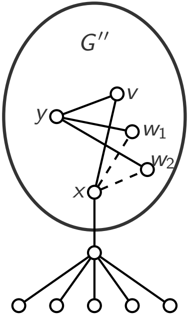

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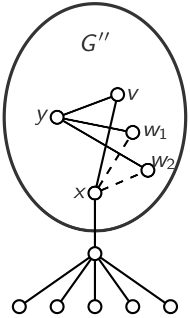
- $G'' \notin \mathcal{F}_{\Delta''}$, has no twins of degree ≥ 2.



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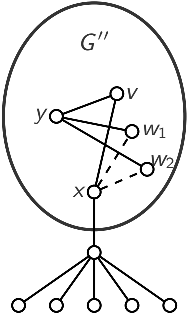

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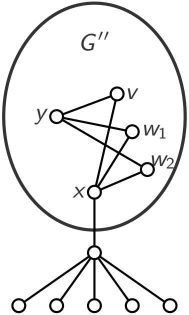
- Put back the deleted edges.



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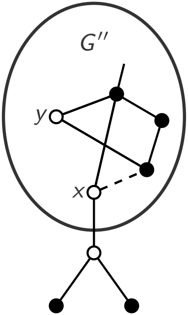
- Most problematic case: x, y not separated and $\Delta(G) = 3$.



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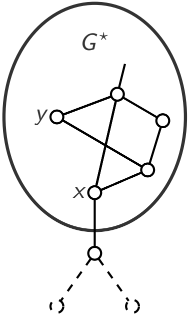
- $G^* = G - \{a, b, y\}$.



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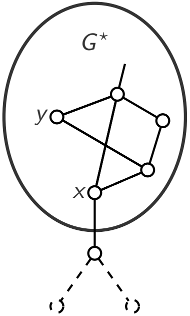

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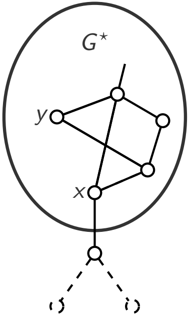
- Most problematic case: x, y not separated and $\Delta(G) = 3$.
- $G^* = G - \{a, b, y\}$.
- G^* is connected, $\notin \mathcal{F}_{\Delta^*}$ and has no twins of degree ≥ 2.
- The result for G holds by induction!



Tightness

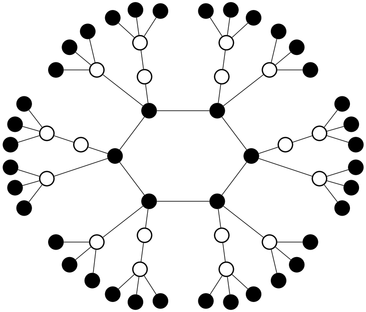


Figure: Graph $G_{6,4}$ with a 6-cycle and maximum degree 4.

Proposition (C., Foucaud, Lehtilä, 2023+)

$$\gamma^{ID}(G_{6,4}) = \frac{\Delta - 1 + \frac{1}{\Delta - 2}}{\Delta + \frac{2}{\Delta - 2}} n.$$

Future work...

Question

Can the general $n - \frac{n}{\Theta(\Delta^3)}$ bound be improved?

Question

What about bipartite grpahs in general?

Question

Or triangle-free graphs?

Thank you!